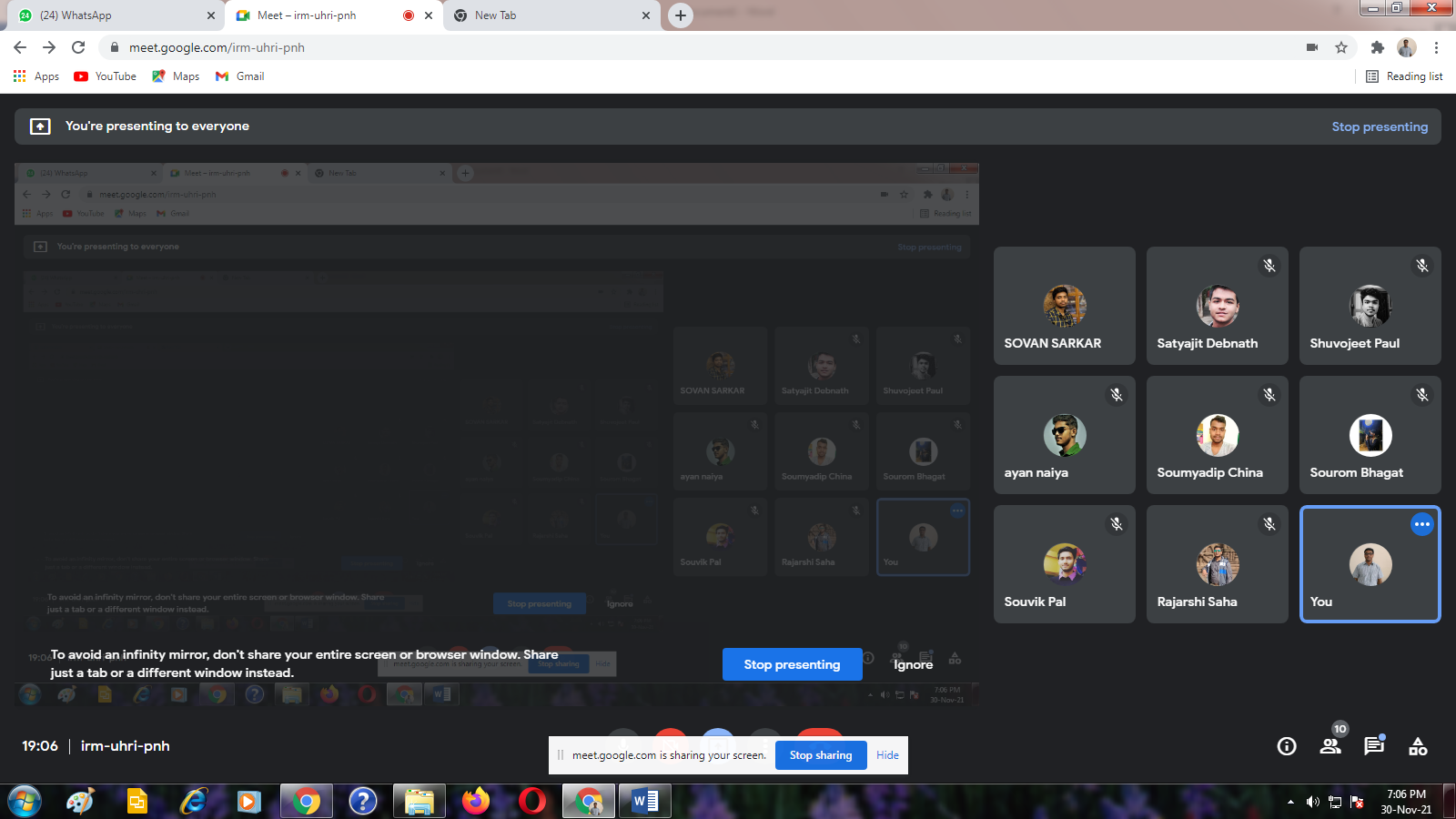
**CLASS 30\_11\_2021**

**UG SEMESTER-3**

**GRAPH THEORY**



**Theorem: A simple graph with n vertices and k components can have at most edges.**

**Proof :**

**k components**

**n vertices**

**=n**

**Maximum edge possible if every component is a complete sub-graph.**

**Number of edges in first component**

**= over i=1 to k**

**(1)**

**We know**

**==n-k**

**Σ(ni-1)=(n-k)**

**Squaring both sides**

**= =**

**or**

**=**

**or**

**=**

**Each component is a valid graph**

**n1>=1,n2>=1,……nk>=1**

**(n1-1)>=0, (n2-1)>=0……..(nk-1)>=0**

**2(n1-1)(n2-1)+2(n2-1)(n3-1)+……… is a positive term**

**+ positive term=**

**or**

**<=**

**or**

**+()+…+ ()<=**

**Or**

**+k<=**

**Or**

**Σni^2-2n+k<=**

**Or**

**(2)**

Putting (2) in (1)

**(proved)**

**Method-2 (Contradiction method…)**

Let us consider a graph with N vertices and k=2 components C1 and C2, C1 having m vertices and C2 with n vertices.

N=m+n

Now m,n>0

Let us consider m>n.

So the components C1 with m vertices has maximum number of edges when it is a simple complete graph with E1=m(m-1)/2 edges.

Similarly maximum number of edges in component C2 with n vertices is

E2=n(n-1)/2

So total no of edges E=E1+E2

=m(m-1)/2+n(n-1)/2

Now let us delete one vertex from C2 and insert it in C1, so C1 has now (m+1) vertices and C2 has (n-1) vertices.

Deleting one vertex from C2 reduces (n-1) edges and adding one vertex in C1 increases m edges in C1. Total increase of edge. m-(n-1)

Now we has considered m>n.

Hence m-(n-1)>0

Hence the edges in new graph increases.

We can proceed like this until n=1. So in this case C1 has m+(n-1) vertices and C2 has 1 vertex. This partition has maximum no of edges.

E=(m+n-1)(m+n-1-1)/2=(N-1)(N-2)/2=(N-k)(N-k+1)/2

Similarly if k=3, then it can be proved that the maximum possible edge will be achieved when one component is having (N-2) vertices and rest of the two has 1 vertices each.

Hence for k components maximum number of edges possible when one components has (N-k+1) vertices and rest of (k-1) components having one vertex each.

Hence maximum no of edges possible

= (n-k+1)(n-k+1-1)/2

=(n-k+1)(n-k)/2 (proved)

problem2-8 prove that a simple graph with n vertices must be connected if it has more than (n-1) (n-2)/2 edges.

We know that a graph with n vertices and k components has maximum number of edges is (n-k)(n-k+1)/2.

A graph is disconnected if it has k=2 or more.

A graph with k=2 has maximum edges E=(n-2)(n-1)/2.

If a graph contains more than (n-2)(n-1)/2 edges than it will be connected.

Problem 2-9: prove that if a connected graph G is decomposed into two sub-graphs g1 and g2, there must be at least one vertex common between g1 and g2.

Problem 2-10: prove that if a connected graph G remains connected after removing an edge e from G, if and only if e is in some circuit.